Cosmic Acceleration With A Positive Cosmological Constant

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We have considered a cosmological model with a phenomenological model for the cosmological constant of the form $\Lambda = \beta \frac{\ddot{R}}{R}$, β is a constant. For age parameter consistent with observational data the Universe must be accelerating in the presence of a positive cosmological constant. The minimum age of the Universe is H_0^{-1}, where H_0 is the present Hubble constant. The cosmological constant is found to decrease as t^{-2}. Allowing the gravitational constant to change with time leads to an ever increasing gravitational constant at the present epoch. In the presence of a viscous fluid this decay law for Λ is equivalent to the one with $\Lambda = 3\alpha H^2$ (α = const.) provided $\alpha = \frac{\beta}{3(\beta-2)}$. The inflationary solution obtained from this model is that of the de-Sitter type.

KEY WORDS: Cosmology, Variable G, Λ, Inflation

1. INTRODUCTION

One of the puzzling problems in standard cosmology is the cosmological constant problem. Observational data indicate that $\Lambda \sim 10^{-55} \text{cm}^{-2}$ while particle physics prediction for Λ is greater than this value by a factor of order 10^{120}. This discrepancy is known as the cosmological constant problem. A point of view which allows Λ to vary in time is adopted by several workers. The point is that during the evolution of the universe the energy density of the vacuum decays into particles thus leading to the decrease of the cosmological constant. As a result one has the creation of particles although the typical rate of the creation is very small. The entropy problem which exists in the Standard Model can be solved by the above mechanism. One of the motivations for introducing Λ term is to reconcile the age parameter and density parameter of the universe with current observational data. Recent observations of Type 1a supernovae which indicate an accelerating universe draw once more the attention to the possible

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existence, at the present epoch, of a small positive cosmological constant (Λ). One possible cause of the present acceleration could be the ever increasing gravitational (constant) forces. As a consequence, a flat universe has to speed up so that gravitational attraction should not win over expansion. Or alternatively, the newly created particles give up their kinetic energy to push the expansion further away.

The purpose of this work is to study the phenomenological decay law for Λ that is proportional to the deceleration parameter. In an attempt to modify the general theory of relativity, Al-Rawaf and Taha [17] related the cosmological constant to the Ricci scalar, $\mathcal{R}$. This is written as a built-in cosmological constant, i.e., $\Lambda \propto \mathcal{R}$. A comparison with our ansatz above for Λ yields a similar behaviour for a flat universe. And since Ricci scalar contains a term of the form $\frac{\ddot{R}}{R}$, one adopts this variation for Λ. We parameterized this as $\Lambda = \beta\frac{\ddot{R}}{R}$, where β is a constant. The cosmological consequences of this decay law are very attractive. This law finds little attention among cosmologists. This law provides a reasonable solution to the cosmological puzzles presently known. We have found that a resolution to these problems is possible with a positive cosmological constant ($\Lambda > 0$). This requires the deceleration parameter to be negative ($q < 0$). Usually people invoke some kind of a scalar field that has an equation of state of the form $p < 0$, p the pressure of the scalar field. A more recent review for the case of a positive cosmological constant is found in [6].

A variable gravitational constant G can also be incorporated into a simple framework in which Λ varies as well, while retaining the usual energy conservation law [1,10,11]. The above decay law leads to a power-law variation for G. Inflationary solutions are also possible with this mechanism thus solving the Standard Model problems. We have recently shown that a certain variation of G may be consistent with palaeontological as well as geophysical data [15].

2. THE MODEL

For the Friedmann–Robertson-Walker metric the Einstein's field equations with the variable cosmological constant and a source term given by a stress-energy tensor of a perfect fluid read

$$3\frac{\dot{R}^2}{R^2} + \frac{3k}{R^2} = 8\pi G\rho + \Lambda \ , \tag{1}$$

$$2\frac{\ddot{R}}{R} + \frac{\dot{R}^2}{R^2} + \frac{k}{R^2} = -8\pi Gp + \Lambda \tag{2}$$

where ρ is the fluid energy density and p its pressure. The equation of the state is taken in the form

$$p = (\gamma - 1)\rho \tag{3}$$

where γ is a constant. From eqs.(1) and (2) one finds

$$\frac{d(\rho R^3)}{dt} + p\frac{dR^3}{dt} = -\frac{R^3}{8\pi G}\frac{d\Lambda}{dt} \tag{4}$$

We propose a phenomenological decay law for Λ of the form [4,5]

$$\Lambda = \beta\frac{\ddot{R}}{R} \tag{5}$$

where β is a constant. Overdin and Cooperstock have pointed out that there is no fundamental difference between the first and second derivatives of the scale factor that would preclude the latter from acting as an independent variable if the former is acceptable [4]. Moreover, from eqs.(1) and (4), one can write

$$\ddot{R} = \frac{8\pi G}{3}\left(1 - \frac{3\gamma}{2}\right)\rho R + \frac{\Lambda}{3}R \ . \tag{6}$$

Thus one example of Λ in the above form is the case when the universe is filled with a fluid characterized by $\gamma = \frac{2}{3}$ and $\beta = 3$. For other values of $\gamma(=1)$, β is not constrained by the Einstein equations, and the general relation

$$\Lambda = \left(\frac{\beta}{\beta - 3}\right)4\pi G\rho \tag{7}$$

shows the ratio of Λ to ρ is constant in this phenomenological model. Now eq.(1) together with eq.(7) yield

$$\Lambda = \left(\frac{\beta}{\beta - 2}\right)H^2. \tag{8}$$

Thus as remarked by Overdin and Cooperstock the model with $\Lambda \propto H^2$ and the above form (eq.(5)) are basically equivalent. We see that in the radiation and in the matter dominated eras the vacuum contributes significantly to the total energy density of the universe in both eras. Thus unless the vacuum always couples (somehow) to gravity such a behavior can not be guaranteed (and understood) at both epochs. Such a mechanism is exhibited in eq.(20). Hence the vacuum domination of the present universe is not accidental but a feature that is present at all times. One would expect that there must have been a conspiracy between the two components in such a way the usual energy conservation holds. Therefore, one may argue that in cosmology the energy conservation principle is not a *priori* principle [17]. We observe from eq.(7) that $\frac{\Lambda_0}{\Lambda_{\text{Pl}}} \approx \frac{\rho_0}{\rho_{\text{Pl}}} \approx \left(\frac{10^{-29}}{10^{93}}\right) = 10^{-122}$, where "0" refers to the present and "Pl" refers to Planck era of the quantity, respectively. Thus such a phenomenological model for Λ could provide a *natural* answer (interpretation) to the puzzling question why the cosmological constant is so small tody, rather than just attributing it to the oldness of our present universe.

For the matter-dominated universe $\gamma = 1$ and therefore eqs.(2),(3), and (5) yield (for $k = 0$)

$$(\beta - 2)\ddot{R}R = \dot{R}^2 \ , \tag{9}$$

which can be integrated to give

$$R(t) = \left(\frac{A(\beta - 3)}{(\beta - 2)} t \right)^{(\beta-2)/(\beta-3)} \quad , \quad \beta \neq 3 \ , \ \beta \neq 2 \ , \tag{10}$$

where $A = $ consatnt. It follows from eq.(5) that

$$\Lambda(t) = \frac{\beta(\beta - 2)}{(\beta - 3)^2} \frac{1}{t^2} \quad , \quad \beta \neq 3 \ . \tag{11}$$

Using eqs.(1), (5) and (10) the energy density can be written as,

$$\rho(t) = \frac{(\beta - 2)}{(\beta - 3)} \frac{1}{4\pi G \ t^2} \quad , \ \beta \neq 3 \tag{12}$$

and the vacuum energy density (ρ_v) is given by

$$\rho_v(t) = \frac{\Lambda}{8\pi G} = \frac{\beta(\beta - 2)}{(\beta - 3)^2} \frac{1}{8\pi G t^2} \quad , \quad \beta \neq 3 \ . \tag{13}$$

The deceleration parameter (q) is defined as

$$q = -\frac{\ddot{R}R}{\dot{R}^2} = \frac{1}{2 - \beta} \ , \ \beta \neq 2 \ . \tag{14}$$

We see from eqs.(11) and (14) that for a positive Λ the deceleration parameter is negative (for $\beta > 2$). For $\beta < 2$ the cosmological constant is negative, $\Lambda < 0$. It has been recently found that the universe is probably accelerating at the present epoch. There are several justifications for this acceleration. Some authors attribute this acceleration to the presence of some scalar field (quintessence field) with a negative pressure filling the whole universe. And this field has a considerable contribution to the total energy density of the present universe. The density parameter of the universe (Ω_m) is given by

$$\Omega_m = \frac{\rho}{\rho_c} = \frac{2}{3} \frac{(\beta - 3)}{(\beta - 2)} \quad , \ \beta \neq 2 \tag{15}$$

where $\rho_c = \frac{3H^2}{8\pi G}$ is the critical energy density of the universe. We notice that the Standard Model formula $\Omega_m = 2q$ is now replaced by $\Omega_m = \frac{2}{3}q + \frac{2}{3}$. However, both models give $q = \frac{1}{2}$ for a critical density. This relation has been found by several authors [1,3].

The density parameter due to vacuum contribution is defined as $\Omega^\Lambda = \frac{\Lambda}{3H^2}$. Employing eqs.(8) this gives

$$\Omega^\Lambda = \frac{\beta}{3(\beta - 2)} \ , \quad \beta \neq 2 \ . \tag{16}$$

We shall define Ω_{total} as

$$\Omega_{\text{total}} = \Omega_m + \Omega^\Lambda \tag{17}$$

Hence eqs.(1), (7), (15) and (16) give $\Omega_{\text{total}} = 1$. This setting is favored by the inflationary scenario. The present value of the age of the universe, deceleration parameter and the cosmological constant are obtained from eqs.(10), (11) and (15)

$$t_0 = \frac{(\beta - 2)}{(\beta - 3)} H_0^{-1}, \quad \Omega_{m0} = \frac{2}{3} \frac{(\beta - 3)}{(\beta - 2)}, \quad \Lambda_0 = \frac{\beta}{(\beta - 2)} H_0^2, \quad \beta \neq 2, \quad \beta \neq 3 . \tag{18}$$

(the subscript '0' denotes the present value of the quantity and H is Hubble constant). Inasmuch as $q < 0$, $\Omega_m < 1$ hence the low density of the universe is no longer a problem. Moreover, in order to solve the age problem we require $\beta > 3$. Thus the constrain $\beta > 3$ may resolve both problems. Observational evidence, however, does not rule out the negative deceleration parameter and the stringent limits on the present value of q_0 are $-1.25 \leq q_0 \leq 2$ [8].

The case $\beta = 2$ represents an empty static universe with $\Lambda = 0$. For $\beta = 0$, the usual expressions for FRW models are recovered. When $\beta = 0$, $\Omega_{m0} = 1$, which is *inconsistent* with many observational tests on scales much too small to be affected by the cosmological constant, e.g. dynamical tests for scales up to a few tens of Mpc. For $\beta = 4$ one obtains $t_0 = 2H_0^{-1}, \Omega_{m0} = \frac{1}{3}, \Omega_0^{\Lambda} = \frac{2}{3}$; $\beta = 12$, $t_0 = \frac{10}{9} H_0^{-1}, \Omega_{m0} = \frac{3}{5}, \Omega_0^{\Lambda} = \frac{2}{5}$. It is interesting to note that when $\beta \to \infty$, all parameters are finite, namely, $R_0 \to t_0$, $t_0 \to H_0^{-1}, \Lambda \to H_0^2$, $\Omega_{m0} \to \frac{2}{3}, \Omega_0^{\Lambda} = \frac{1}{3}$. Thus the values of β which are consistent with

$$\Omega_{m0} = 0.3 \pm 0.1$$

are

$$\beta = 4.0 \pm 0.5$$

but the ages are very high. For instance, with a high, but still consistent with observational constraints, value of the Hubble constant, $H_0 = 80$ km/s/Mpc, and a rather generous,

$$t_0 = 15 \pm 2 \ \text{Gyr}$$

then values of β consistent with this are:

$$5.5 < \beta \leq 19 .$$

This suggests a best fit value somewhere around $\beta = 5$, which would give the somewhat high matter density of $\Omega_{m0} = 0.44$ and rather high age of $t_0 = 18.3$ Gyr. We have recently [15] investigated the implications of a variable G on the Earth-Sun system, contrary to what have been believed, we have found that the palaeontological data are consistent with a variable G provided that the age of the universe is $t_0 \sim 11 \times 10^9$ years and that $G \propto t^{1.3}$. However, a recent value for the age of the universe from

gravitational lensing suggests $t_0 \sim 11 \times 10^9$ years. Recent estimates from observations of galaxy clustering and their dynamics indicate that the mean mass density is about one-third of the critical value [9]. Thus if $\beta \neq 0$ then the present age of the universe can not be less than H_0^{-1}. This constrain represents our strongest prediction for the age of the Universe.

3. A MODEL WITH VARIABLE G

We now consider a model in which both G and Λ vary with time. Imposing the usual energy conservation law one obtains [1,10,11]

$$\dot{\rho} + 3\gamma H \rho = 0 \ , \tag{19}$$

and

$$\dot{\Lambda} + 8\pi \dot{G} \rho = 0 \ . \tag{20}$$

Using eqs.(10) and (11), eqs.(19) and (20) yield

$$\rho(t) = Dt^{-3(\beta-2)/(\beta-3)} \ , \quad D = \text{const.} \tag{21}$$

and

$$G(t) = \left(\frac{(\beta-2)}{4\pi A(\beta-3)} \right) t^{\beta/(\beta-3)} \ \beta \neq 3 \ . \tag{22}$$

For $\beta = 0$, G=const. and $\rho = Dt^{-2}$ and $R = \left(\frac{3}{2}At\right)^{2/3}$, which is the usual FRW result. Clearly for $\beta > 3$ the gravitational constant increases with time while for $\beta < 3$ it decreases with time. Once again the constraint $\beta > 3$ considered before implies that the gravitational constant increases with time. An increasing gravitational constant is considered by several workers [1,10,11,14].

In a recent work we have shown that the variation of the gravitational constant is consistent with palaeontological data [15]. The gravitational constant might have had a very different value from the present one. This depends strongly on the value of β assumed at a given epoch. The development of the large-scale anisotropy is given by the ratio of the shear σ to the volume expansion $(\theta = 3\frac{\dot{R}}{R})$ which evolves as [16]

$$\frac{\sigma}{\theta} \propto t^{(3-2\beta)/(\beta-3)} \ , \tag{23}$$

and since $\beta > 3$ this anisotropy decreases as the universe expands and this explains the present observed isotropy of the Universe.

For example, if $\beta = \frac{3}{2}$ then $G \propto t^{-1}, R \propto t^{1/3}, \rho \propto t^{-1}$. This behavior of G was considered by Dirac in his Large Number Hypothesis (LNH) model [12]. In an earlier work we have shown that some non-viscous

models are equivalent to bulk viscous ones. This behavior is also manifested in our present model provided one takes $\beta = \frac{3(2n-1)}{(3n-2)}$, where the bulk viscosity (η) is defined as $\eta = \text{const.}\rho^n$, where $0 \leq n \leq 1$ [1]. The two models, though different in the form of Λ, are equivalent if one puts $\alpha = \frac{\beta}{3(\beta-2)}$. Thus the decay law $\Lambda = \beta\frac{\ddot{R}}{R}$ and $\Lambda = 3\alpha H^2$, where $\alpha = \text{const.}$, are identical in the presence of a cosmic fluid.

4. AN INFLATIONARY SOLUTION

This solution is obtained from eqs.(1), (2), (3) and (5) with $\beta = 3$. One then gets $R\ddot{R} = \dot{R}^2$, which integrated to give

$$R = \text{const.} \exp(Ct) \tag{24}$$

where $H = C = \text{const.}$. Applying eq.(24) to eqs.(5) and (20), employing eq.(1), we get

$$\Lambda = 3H^2 \;\;,\;\; \rho = 0 \;\;,\;\; G = \text{const.} . \tag{25}$$

This is the familiar de-Sitter inflationary solution (in the matter dominated epoch). A similar inflationary solution is obtained with $\beta = 3$ in the radiation dominated epoch. Inflationary models employ a scalar field (inflaton) to arrive at this solution. These solutions help resolve several cosmological problems associated with the standard model (flatness, horizon, monopole, etc.) The inflationary solution resolves some of the outstanding issues of standard cosmology.

5. CONCLUSION

In this paper we have considered the cosmological implications of a decay law for Λ that is proportional to $\frac{\ddot{R}}{R}$. The model is found to be very interesting and apparently a lot of problems can be solved. To solve the age parameter and the density parameter one requires the cosmological constant to be positive or equivalently the deceleration parameter to be negative. This implies an accelerating universe. However, the strongest support for an accelerating universe comes from intermediate redshift results for Type 1a supernovae. The model predicts that the minimum age of the Universe is H_0^{-1}. The behavior that $\Lambda \propto t^{-2}$ is found by several authors. The gravitational constant is found to increase with time at the present epoch. Our model predicts an inflationary phase in the matter dominated epoch as well as in the radiation dominated epoch. The cosmological tests for this model can be obtained from those ones already investigated by us [7]. The choice among these models awaits the emergence of the new data.

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